Question 1. [7 marks]

$f(x) = e^{x}$ and $g(x) = 3x + 1$

a. Determine:

(1) $f_0 g(-1)$ [1]

(2) $g_0 f(-1)$ [1]

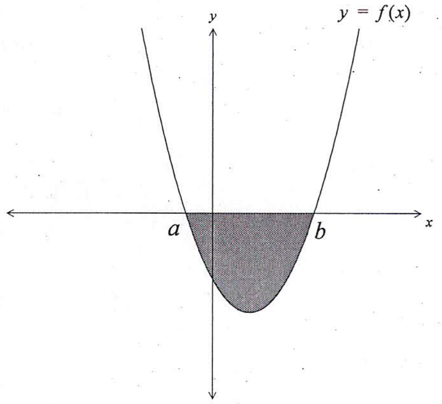
(3) $g_0 f(x)$ [1]

b. State the domain and range of $g[f(x)]$ [2]

c. Describe the geometrical effect(s) when $f(x)$ is transformed to $f[g(x)]$. [2]

Question 2. [6 marks]

a. The shaded region below has an area of 5 square units.



Evaluate $\displaystyle\int_a^b 3f(x)\ dx$ [2]

b. $\displaystyle\int_p^q 1\ dx = 7$ and $\displaystyle\int_1^p x\ dx = 4$

Determine the values of p and q. [4]

Question 3. [7 marks]

$$f(x) = -x^2 + 4x + 5$$

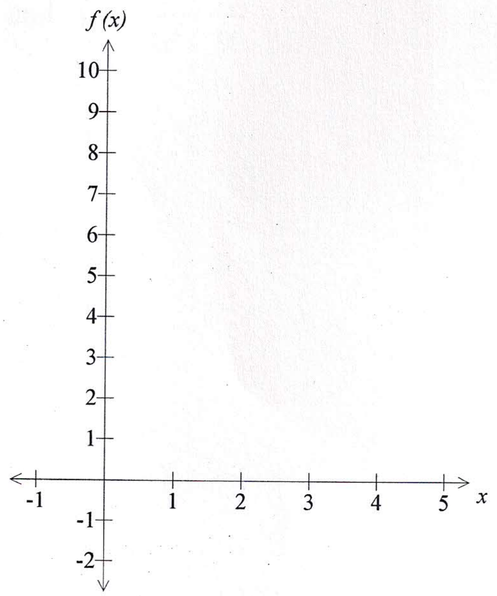
a. Evaluate:

(1) $f(2)$ [1]

(2) $f'(2)$ [1]

(3) $f''(2)$ [1]

b. (1) Sketch the graph of $f(x)$ over the domain $-1 \le x \le 5$ on the axes provided. [1]



(2) With reference to your sketch, explain the significance of each answer from part a.. [3]

Question 4. **[6 marks]**

a. Simplify $\dfrac{4x + 12}{x^2 - 9}$, stating any exclusions from the domain. [2]

Hence, make use of the chain rule with Leibnitz notation, to determine:

b. $\dfrac{dz}{dy}$, if $z = \dfrac{1}{3x}$ and $y = \dfrac{4x + 12}{x^2 - 9}$ [4]

Question 5. **[9 marks]**

A rectangle with fixed area, A, has perimeter, P, where $P = 2x + \dfrac{128}{x}$.
The length of one of the sides of the rectangle is x metres.

a. If x increases from 2 metres to 2.01 metres, determine the associated change in P, by using the Incremental Formula (small change). [4]

b. Determine the value of x such that P is a minimum.
Show using Calculus, that P is the minimum. [3]

c. Determine the dimensions of the rectangle from Part b. [1]

d. Determine the area of the rectangle from Part b. [1]

Question 6. [3 marks]

There are eight black, seven white and three grey horses for sale. Mary buys three horses at random at the sale.

Calculate the probability that she purchases one of each colour, giving your answer as a fraction in its simplest form. [3]

Question 7. [2 marks]

Solve the following system of equations.

$$\begin{aligned} x + y + z &= 6 \\ 2x - y - z &= -3 \\ 3x + 2y + 5z &= 22 \end{aligned}$$

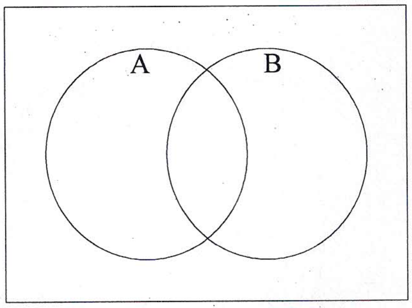
[2]

End of Section One

Question 1. [7 marks]

For events A and B represented in the Venn diagram below:

$P(A \cup B) = 0.7$, $P(A) = 0.4$ and $P(A \cap B) = 0.1$



a. **Calculate:**

(1) $P(B)$ [1]

(2) $P(A' \cap B)$ [1]

(3) $P(B \mid A)$ [2]

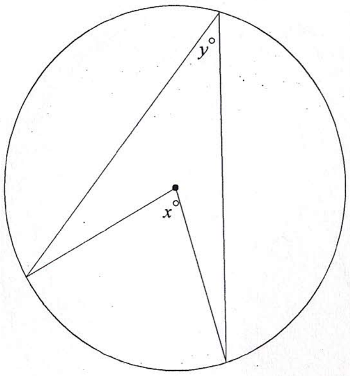
b. Determine whether events A and B are mutually exclusive. Explain your answer. [1]

c. Are events A' and B independent? Justify your answer. [2]

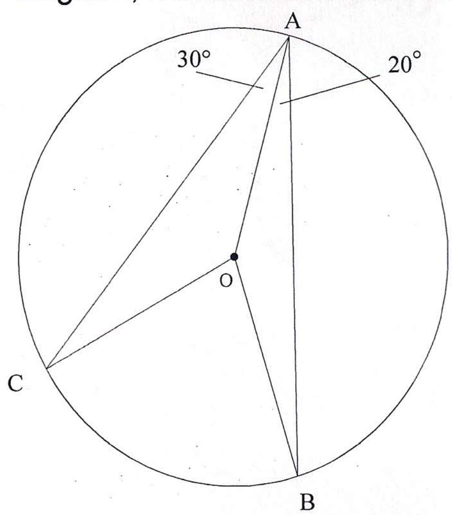
Question 2. **[5 marks]**

There is a theorem called the Central Angle Theorem, which states that the angle at the centre of a circle is twice the angle at the circumference, standing on the same arc.

In the diagram below, $x = 2y$.



A student proof of the theorem is as follows.
For this particular diagram, measurements are not to scale.



Since $\quad AO = OC$, then $\angle C = 30° \quad \therefore \angle AOC = 180 - (30 + 30) = 120°$

Since $\quad AO = OB$, then $\angle B = 20° \quad \therefore \angle AOB = 180 - (20 + 20) = 140°$

$\therefore \quad \angle COB = 360 - (120 + 140) = 100°$

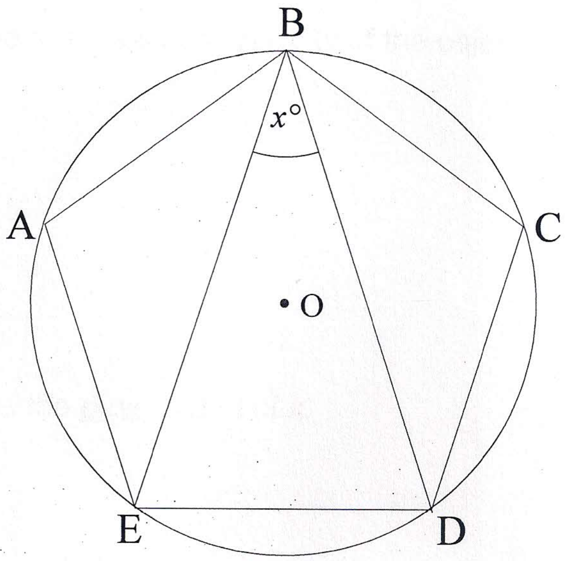
$\therefore \quad \angle COB = 2\angle CAB$

$\therefore \quad x = 2y$

a. Discuss the validity of the student's proof. [2]

Question 2 continued.

b. ABCDE is a regular pentagon inscribed in a circle.
Show clearly how to use the central angle theorem to calculate the size of x. [3]



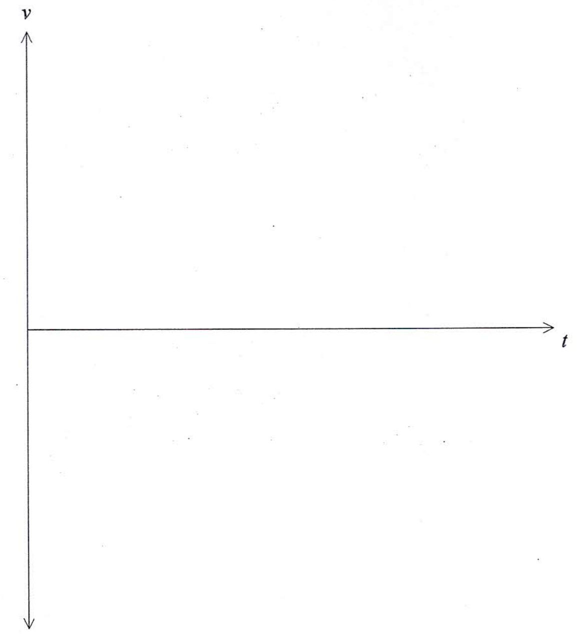
Question 3. **[8 marks]**

An object moves along a straight line, passing through point O with a velocity of 4 m/s. It has an acceleration, a ms^{-2}, given by $a = 16 - 2t$, t seconds after passing O. The object comes to rest when $t = k$ seconds.

a. Determine the maximum velocity of the object. [2]

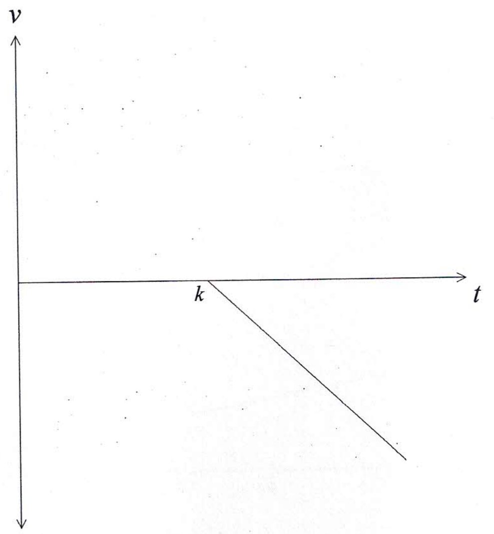
b. Determine the *exact* value of k. [2]

c. Sketch the v–t graph on the axes below, showing and labelling all key points. [2]



Question 3 continued.

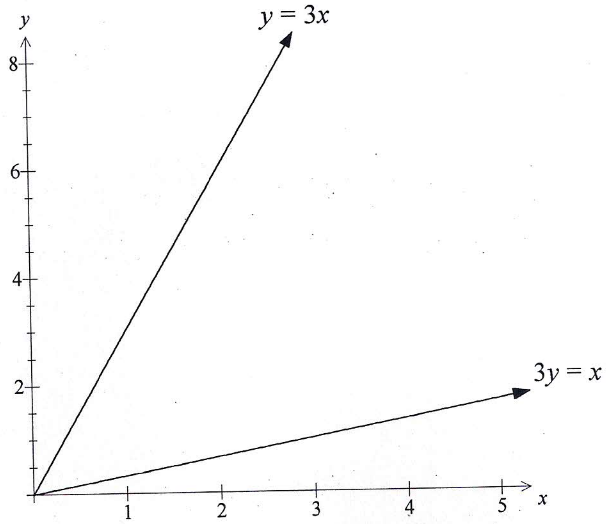
After coming to rest, the object travels for a further three seconds with a velocity given by $v = n - mt$, where $t > k$. This is represented on the axes given below.



d. State the expression, in terms of k, which represents the distance travelled in that three second period. [2]

Question 4. [5 marks]

The following questions relate to this diagram.



a. Shade the region satisfying the simultaneous inequalities;

$y \le 3x \cap 3y \ge x \cap x \le 2$ [2]

b. If x and y are integers, complete the solutions, S, to the inequalities in (a). [1]

$S = \{ (0, 0), (1, 1), (1, 2), (1, 3), (2, 6), (2, 5),$

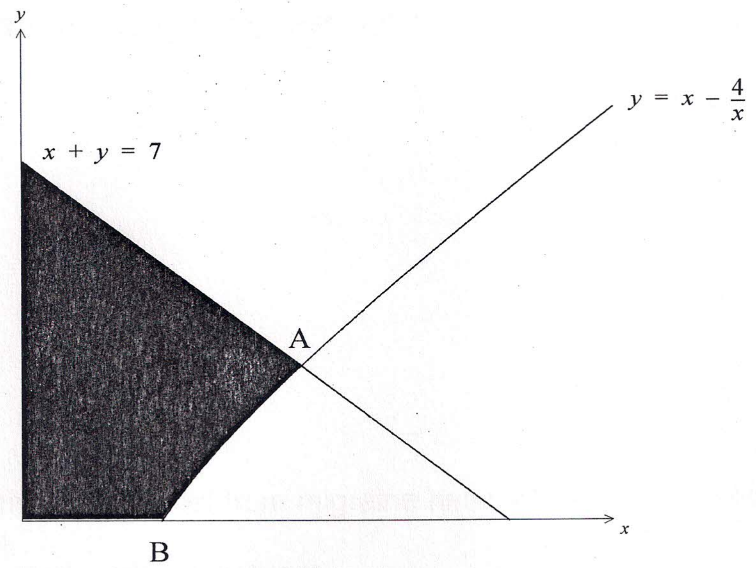
c. (1) What is the probability that an integer solution, chosen at random from S maximises the function $F = kx$? ($k > 0$) [1]

(2) Consider the function $G = kx + my$, where k and m are positive. What is the maximum value of G, in terms of k and m, over the region described in Part a? [1]

Question 5. [4 marks]

A spherical balloon is being inflated by pumping gas into it at a rate of 5 m^3/minute. Determine the rate at which the diameter is increasing when the radius is 1 m. [4]

Question 6. [6 marks]



The shaded area is bounded by the x and y axes, the line $x + y = 7$ and the curve with equation $y = x - \frac{4}{x}$.

a. Determine the co-ordinates of A and B. [1]

b. State an expression, which when evaluated will determine the area of the shaded region. Give the area correct to two decimal places. [2]

c. The shaded region is revolved 360^0 around the x axis.

(1) Write down the expression for the volume of the solid generated. [2]

Question 6 continued

(2) Determine the volume generated exactly. [1]

Question 7. **[7 marks]**

"No–ache" claims to give relief from migraine headaches. It is claimed to be effective in 90% of cases.
Trials are conducted with four patients.

a. If X is the number of patients obtaining relief, then assuming the claim is correct,

(1) why is X a binomial random variable? [2]

(2) determine the mean and standard deviation of X. [2]

(3) determine the probability that none of the four patients gains relief. [1]

b. If none of the four gets relief, do you think that the 90% claim is reasonable? Explain on the basis of the probabilities involved. [2]

Question 8. [14 marks]

A trout farm is selling all its current stock of adult trout, kept in a huge tank.
It is estimated that there are 10 000 trout in the tank. Weights of trout in the tank are thought to be normally distributed with a mean of 1 kg, and a standard deviation of 0.2 kg.

a. Determine the percentage of trout weighing between 0.9 kg and 1.2 kg. [2]

Grade A trout weigh more than one standard deviation above the mean.

b. How many grade A fish are in the tank? [2]

The smallest 10% (by weight) of trout are used for fertiliser.

c. What is the largest weight for such a trout? [2]

The manager has export orders totalling 4 tonnes of trout weighing between 0.9 and 1.2 kg.

d. Determine whether the orders can be filled. Explain your decision. [2]

Question 8 continued.

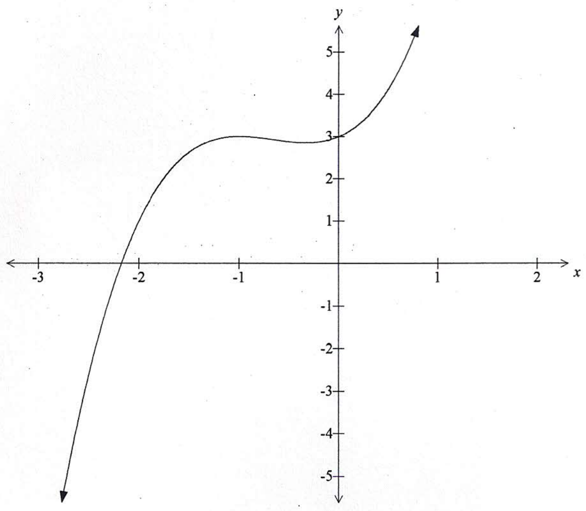
e. A sample of 25 trout are randomly selected. They have a mean of 1.04 kg. Determine a 99% confidence interval for the population mean. [2]

f. How large a sample should be taken to ensure, with 95% confidence, that the sample mean is within 100 g of 1 kg? [2]

g. Six trout are selected at random from the tank. What is the probability that exactly four of them will weigh between 0.9 kg and 1.2 kg? [2]

Question 9. [9 marks]

The graph of $y = x^3 + 2x^2 + x + 3$ is shown.



a. Use the second derivative to show that a possible point of inflection exists at $x = -\frac{2}{3}$. [2]

b. Use a sign test to verify that the point where $x = -\frac{2}{3}$ is, in fact, a point of inflection. [2]

Question 9 continued.

c. Calculate the equation of the tangent to the curve drawn at the point of inflection. [2]

d. A conjecture is made that a tangent drawn through a point of inflection will go through at least one turning point.
Use a counterexample to show that this conjecture is false. [3]

Question 10. [6 marks]

Two digit numbers are divisible by three if the sum of the digits is divisible by three.
eg. 81 is divisible by three; 8 + 1 = 9 which is divisible by three.

a. Test this statement on a two digit number of your choosing. [1]

The statement can be proven using Algebra, as follows:
"If the two digit number is *ab*, then the sum of the digits is $a + b$. If that sum is divisible by three, then $a + b = 3k$, where k is a whole number".

The value of *ab* is $10a + b$.

$$\begin{aligned} \therefore \quad 10a + b &= 9a + a + b \\ &= 9a + 3k \\ &= 3(3a + k) \end{aligned}$$

$\therefore \quad 10a + b$ is divisible by three.

b. Prove that the statement is true for three digit numbers.
ie. If $a + b + c$ is divisible by three, prove that *abc* is divisible by three. [3]

c. John believes that if the sum of the digits in a four digit number is divisible by five, then the four digit number must be divisible by five.

(1) Find an example that confirms this statement. [1]

(2) Find a counter example. [1]

Question 11. [4 marks]

A population, y, increases according to the differential equation:

$$\frac{dy}{dt} = 0.04\,y \quad \text{where } t \text{ is the time, in years, after the start of 2000}$$

The population at the start of 2000 has size 1 000.

a. State the equation for population, y, in terms of t. [1]

b. State the population size when $t = 5$. [1]

c. Determine the doubling time for the population. [2]

Question 12. [5 marks]

The time, T minutes, that Erica spends at her local coffee shop has a mean of 30 and a standard deviation of 20.

a. It is <u>*not*</u> likely that the distribution of T is normal. Explain why. [1]

$\overline{T}$ represents the mean time of a random sample of 40 of Erica's visits.

b. (1) Even though T is not normally distributed, the distribution of $\overline{T}$ will be normally distributed. Explain why. [1]

(2) Calculate the probability that $\overline{T}$ is more than 25 minutes. Show working. [3]

End of Section Two